ordinate; u, velocity component in the x direction; v, velocity component in the y direction; x, distance along the axis; y, distance from the free surface of film; y', distance from wall; α, heat-transfer coefficient; δ, film thickness; λ, thermal conductivity; μ, dynamic viscosity; ρ, density; η, shear stress. Subscripts: c, cooling medium; eff, effective properties; f, film; G, gas; I, free film surface; L, liquid; s, saturation line; t, turbulent component; w, wall.

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INFLUENCE OF VARIABLE VISCOSITY ON THE HEAT TRANSFER IN A LAMINAR FLUID FILM

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An expression taking account of the heat flux direction during heat transfer in a laminar fluid film is obtained from the approximate solution of the equations of motion and heat conduction.

The influence of a temperature change in the viscosity across the layer is not taken into account in the majority of papers [1-4] examining heat transfer in fluid films although experimental results on the cooling, heating, and also the change in the temperature drop yield different results. Some authors [5] use the factor $(\nu_f/\nu_w)^{0.25}$ by analogy with heat transfer in pipes [6] or on the basis of the Bays experiments [7] conducted in short tubes.

We attempted to estimate the influence of variable viscosity on heat transfer on the basis of an analytical solution of the fundamental equations.

Stable two-dimensional flow of a laminar fluid layer along a vertical wall with a semi-infinite heating section is considered. The Ox axis of the coordinate system is on the solid boundary in the flow direction, while the Oy axis is perpendicular to the stream and the wall. For $x < 0$ the wall temperature is t_0, while for $x \geqslant 0$ it is given by a smooth function $t_w(x)$. The change in the kinetic viscosity coefficient $\nu(t)$ is approximated by a hyperbola. Heat

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liberation due to friction and the changes in ρ, p, λ with the temperature are considered negligible.

For $x \geqslant 0$ the equations or motion and heat conduction for a laminar layer are written as follows in dimensionless form

$$0 = \frac{1}{h^2 R_0} \frac{\partial}{\partial \eta} \left[N(T) \frac{\partial u}{\partial \eta} \right] + \frac{3}{R_0}; \tag{1}$$

$$\frac{h^2}{3} U \frac{\partial T}{\partial \xi} = \frac{\partial^2 T}{\partial \eta^2}. \tag{2}$$

Here

$$\xi = x/3\,\mathrm{Pe}\,\delta_0;\ \eta = y/\delta_0 h;\ \delta_0 = (3\nu_0 \Gamma/g)^{1/3};\ U = u/u_0;$$

$$h = \delta/\delta_0;\ u_0 = \Gamma/\delta_0;\ T(\xi, \eta) = (t - t_0)/\Delta t;\ \theta(\xi) = (t_w - t_0)/\Delta t;$$

$$\Delta t = \max|t - t_0|;\ N(T) = \nu(t)/\nu_0 = [1 + b(t - t_0)]^{-1} = (1 + BT)^{-1};$$

$$B = b\Delta t;\ \mathrm{Re}_0 = 4\mathrm{R}_0 = 4\Gamma\nu_0^{-1};\ \mathrm{Pe} = \Gamma a^{-1}.$$

The boundary conditions take account of the absence of tangential stresses and heat flux on the free surface

$$U(\xi, 0) = 0;\ \frac{\partial U(\xi, 1)}{\partial \eta} = 0; \tag{3}$$

$$T(\xi < 0, 0) = 0;\ T(\xi \geqslant 0, 0) = \theta(\xi);\ \frac{\partial T(\xi, 1)}{\partial \eta} = 0. \tag{4}$$

The equation of motion (1) is integrated in general form with (3) taken into account

$$U(\xi, \eta) = 3h^2 \int_0^\eta \frac{(1-\eta)}{N(T)} d\eta = 3h^2 \int_0^\eta (1-\eta)(1 + BT)\, d\eta. \tag{5}$$

Substituting the velocity profile (5) into (2), we obtain the equation

$$h^4 \frac{\partial T}{\partial \xi} \int_0^\eta (1-\eta)(1 + BT)\, d\eta = \frac{\partial^2 T}{\partial \eta^2}. \tag{6}$$

In order to approximate this problem to the Nusselt formulation [8], we assume the function $\theta(\xi)$ to vary smoothly between 0 and $\bar{\theta} = \pm 1$ (heating—cooling) in the section $0 < \xi < \Delta_0$, where Δ_0 can be sufficiently small.

We represent the solution of (6) in the form of a power series in η:

$$T(\xi, \eta) = \theta(\xi) + \sum_{i=1}^{\infty} \varphi_i(\xi)\eta^i. \tag{7}$$

Substituting (7) into (6) and equating coefficients of identical powers of η, we obtain a general expression for the term φ_i of the series (7) for $\xi > \Delta_0$:

$$\varphi_i(\xi) = \frac{h^4}{i(i-1)} \left[(1 + B\bar{\theta}) \left(\varphi'_{i-3} - \frac{1}{2} \varphi'_{i-4} \right) + B \left(\sum_{l=1}^{i-4} \frac{\varphi_l \varphi'_{i-l-3}}{l+1} - \sum_{l=1}^{i-5} \frac{\varphi_l \varphi'_{i-l-4}}{l+2} \right) \right]. \tag{8}$$

The first terms of series (7) can be expressed in terms of φ_1:

$$\varphi_2 = 0;\quad \varphi_3 = 0;\quad \varphi_4 = \frac{h^4}{12}(1 + B\bar{\theta})\varphi'_1;\ \varphi_5 = \frac{h^4}{40}\varphi'_1 [B\varphi_1 - (1 + B\bar{\theta})];\quad \varphi_6 = \frac{h^8}{90} B\varphi_1\varphi'_1;$$

$$\varphi_7 = \frac{h^8}{504}(1 + B\bar{\theta})^2 \varphi''_1;\ \varphi_8 = h^8 \left[-\frac{\varphi''_1}{840}(1 + B\bar{\theta})^2 + \varphi_1\varphi''_1 \frac{B(1 + B\bar{\theta})}{840} + \varphi'^2_1 B \frac{(1 + B\bar{\theta})}{1344} \right]. \tag{9}$$

Here and henceforth the prime denotes differentiation with respect to ξ. For series (7) the second of the boundary conditions (4) goes over into

$$\sum_{i=1}^{\infty} i\varphi_i(\xi) = 0. \tag{10}$$

We substitute (9) here and obtain an ordinary differential equation for the heat flux on the solid surface φ_1:

$$\varphi_1'' + \frac{525}{11h^4(1+B\bar{\theta})}\varphi_1' + \frac{2520}{11h^8(1+B\bar{\theta})^2\varphi_1} + \frac{147\ B}{11h^4(1+B\bar{\theta})}\varphi_1\varphi_1' + \frac{24\ B}{11(1+B\bar{\theta})}\varphi_1\varphi_1'' + \frac{15\ B}{11(1+B\bar{\theta})}\varphi_1'^2 = 0. \tag{11}$$

The solution φ_{10} for the linear part of (11) has the form

$$\varphi_{10} = C_{10}\exp\left[-\frac{d_1(\xi-\Delta_0)}{h^4(1+B\bar{\theta})}\right] + C_{20}\exp\left[-\frac{d_2(\xi-\Delta_0)}{h^4(1+B\bar{\theta})}\right].$$

Here $d_1 = 5.412$ and $d_2 = 42.31$. The dimensionless layer thickness is assumed to be a slowly varying function. Taking into account that the coefficients of the nonlinear terms depend on B and have a noticeably lesser value than for the linear terms, it can be assumed that the correction to the heat flux for the nonlinearity $\varphi_{11} = \varphi_1 - \varphi_{10}$ is considerably less than φ_{10}. It can be shown that $d_j \to 2\beta_j^2$ is satisfied as the number of terms taken into account in the series (7) increases, where β_j are the eigenvalues of the linear Gertz problem for a plane tube with boundary conditions of the first kind [6, 9], whose formulation is identical to the linear problem of heat transfer in a fluid layer in the absence of heat transport through the free surface. Therefore

$$\varphi_{10} = \sum_{j=1}^{\infty} C_{j0}\exp\left[-\frac{2\beta_j^2(\xi-\Delta_0)}{h^4(1+B\bar{\theta})}\right]. \tag{12}$$

The coefficients C_{j0} remain undetermined but should equal the corresponding coefficients of the linear problem as $B \to 0$.

In estimating the convergence of series (7) it should be taken into account that the signs of φ_1 and $\bar{\theta}$ as well as of φ_1' and φ_1 are opposite; therefore, the terms of series (7) form a sign-varying group. The sign of the term hence depends on the relationship between the components; the general expression (8) for φ_i is also a sign-varying series. Then its magnitude can be estimated by means of the greatest component. Moreover, since the order of the quantities h, B, $|C_{j0}|$ does not exceed one, then by using (8) and (12) we obtain the following estimates

$$|\varphi_i| < \left|\frac{\varphi_{i-3}'}{i(i-1)}\right| < \frac{|\varphi_1^{(m)}|}{i(i-1)(i-3)(i-4)(i-6)\ldots 1} < \frac{\sum^{j}(2\beta_j^2)^m C_{j0}}{(2k)!},$$

where m = [i/3]; k = [i/2] (here [x] is the integer part of the number).

In connection with the fact that solutions with finite β_j, for which the majorizing series $\sum_{j=1}^{N}(2\beta_j^2)^m C_{j0}(2k)!^{-1}$ converges absolutely, have physical meaning, the series (7) is convergent.

For $\xi > \xi_0 = \Delta_0 + 0.055$, φ_{10} is described with sufficient accuracy by an exponential with the minimum eigenvalue β_j:

$$\varphi_{10} \simeq C_{10}\exp[-A(\xi-\Delta_0)];\ A = \frac{2\beta_1^2}{h^4(1+B\bar{\theta})}$$

Substituting the first six terms of the series (7) into (9), we obtain for φ_1:

$$\varphi_1' + p_1\varphi_1 + \varphi_1\varphi_1' q_1 = 0. \tag{14}$$

Here $p_1 = 4.8h^{-4}(1 + B\bar{\theta})^{-1}$ and $q_1 = 0.28B(1 + B\bar{\theta})^{-1}$.

According to the principles mentioned earlier p_1 must be replaced by A, then the linearized equation for φ_{11} becomes

$$\varphi_{11}' + p_2\varphi_{11} = q_2, \qquad (15)$$

where

$$p_2 = \frac{p_1(1 - Aq_1\varphi_{10})}{1 + q_1\varphi_{10}}; \quad q_2 = \frac{Aq_1\varphi_{10}^2}{1 + q_1\varphi_{10}}.$$

The solution of (15) has the form

$$\varphi_{11}|_{\xi>\xi_0} = \left[\int\limits_{\xi_0}^{\xi} q_2 \exp\left(\int\limits_{\xi_0}^{\xi} p_2 d\xi\right) d\xi + \varphi_{11}(\xi_0)\right] \exp\left(-\int\limits_{\xi_0}^{\xi} p_2 d\xi\right) = \varphi_{10}\left[\left(\frac{\varphi_{11}(\xi_0)}{\varphi_{10}(\xi_0)} + \frac{1}{2}\right)\frac{(1 + q_1\varphi_{10}(\xi_0))^2}{(1 + q_1\varphi_{10})^2} - \frac{1}{2}\right]. \qquad (16)$$

Here $\varphi_{10}(\xi_0) = C_{10}\exp[-A(\xi_0 - \Delta_0)]$. The quantity $\varphi_{11}(\xi_0)$ is a correction to the heat flux for the nonlinearity at $\xi = \xi_0$, it cannot be determined within the framework of this approximation.

It can be shown from the conditions for formulation of the problem that the mean temperature of the layer at a distance ξ from the origin is

$$T_m(\xi) = -\frac{3}{h}\int\limits_0^{\xi}\varphi_1 d\xi = D - \frac{3}{h}\int\limits_{\xi_0}^{\xi}\varphi_1 d\xi = \bar{\theta} + \frac{3\varphi_{10}}{Ah}\left[\frac{1}{2} + \left(\frac{1}{2} + \frac{\varphi_{11}(\xi_0)}{\varphi_{10}(\xi_0)}\right)\frac{(1 + q_1\varphi_{10}(\xi_0))^2}{(1 + q_1\varphi_{10})}\right]. \qquad (17)$$

It was taken into account in the calculations that $T_m \to \bar{\theta}$ as $\xi \to \infty$. The local Nusselt number is defined as [6]

$$\mathrm{Nu}(\xi) = \frac{\alpha\delta_0}{\lambda} = -\frac{\varphi_1}{\bar{\theta} - T_m(\xi)}. \qquad (18)$$

The mean Nusselt number is introduced as follows:

$$\langle \mathrm{Nu}(\xi)\rangle = \frac{\langle\alpha\rangle\delta_0}{\lambda} = \frac{1}{\xi}\int\limits_0^{\xi}\mathrm{Nu}(\xi)\,d\xi = -\frac{1}{3\xi}\ln\frac{\bar{\theta} - T_m(\xi)}{\bar{\theta} - T_m(0)} \qquad (19)$$

$$= \frac{2\beta_1^2}{3h^4(1 + B\bar{\theta})} - \frac{1}{3\xi}\ln W(\xi) = \frac{1,88}{h^4(1 + B\bar{\theta})} - \frac{1}{3\xi}\ln W,$$

where

$$W = -\frac{3C_{10}}{Ah\bar{\theta}}\left\{\frac{1}{2} + \left(\frac{1}{2} + \frac{\varphi_{11}(\xi_0)}{\varphi_{10}(\xi_0)}\right)\frac{(1 + q_1\varphi_{10}(\xi_0))^2}{(1 + q_1\varphi_{10})}\right\}.$$

It is here taken into account that $T_m(0) = 0$ as well as that the signs of φ_{10} and $\bar{\theta}$ are opposite. An analysis of the expression $W(\xi)$ shows that the quantity W does not exceed 2 as the parameter B varies between sufficiently broad limits, even if it is admitted that $\varphi_{11}(\xi_0) \simeq \varphi_{10}(\xi_0)$. The component, including the logarithm, can therefore be considered negligible (to 5%) for $\xi > 5$ or $x > 15\mathrm{Pe}\delta_0$.

The formula for $\langle \mathrm{Nu}(\infty)\rangle$ as $\xi \to \infty$ differs from the result obtained by Nusselt [1] by the factor $h^{-4}(1 + B\bar{\theta})^{-1}$. Let us note that it follows from the formula for the layer thickness $\delta = (3\nu\Gamma/g)^{1/3}$ that $h \to (1 + B\bar{\theta})^{-1/3}$ as $\xi \to \infty$. Then

$$\langle \mathrm{Nu}(\infty)\rangle = \frac{1,88}{(1 + B\bar{\theta})^{1/3}} = 1,88\left(\frac{\nu_0}{\nu_w}\right)^{1/3}. \qquad (20)$$

The thickness of the isothermal flux δ_f at some mean temperature t_f can be selected as the unit of length, then $\delta_f = (3\nu_j\Gamma/g)^{1/3}$ from which $\delta_f/\delta_0 = (\nu_j/\nu_0)^{1/3}$. After renormalization, the Nusselt number $\langle \mathrm{Nu}(\infty)\rangle$ equals

$$\langle \mathrm{Nu}(\infty)\rangle_f = \frac{\langle\alpha\rangle\delta_f}{\lambda} = 1,88\left(\frac{\nu_f}{\nu_w}\right)^{1/3}. \qquad (21)$$

If the wall temperature $t_w(\infty)$ is taken as governing, then the factor in the right-hand side of (21) equals 1, i.e., $\langle \mathrm{Nu}(\infty)\rangle_w = 1.88$. Therefore, the average heat-transport intensity on the heating section $0 < \xi < 5$ has the same value with the variable viscosity taken into account as for heating of a fluid with unchangeable properties with a layer thickness

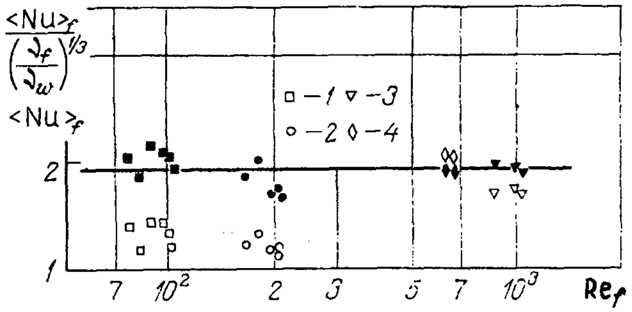

Fig. 1. Comparison of the mean Nusselt number $\langle Nu(\infty)\rangle_f$ (open points) and the quantities $\langle Nu(\infty)\rangle_f(\nu_f/\nu_w)^{1/3}$ (blackened points) for laminar and pseudolaminar flows of aqueous glycerine solutions of different concentrations [10] and water [11]: 1) 82.9; 2) 72.3; 3) 14%; 4) water.

which it takes on as $\xi \to \infty$ [8]. However, the use of (21) is more convenient in that all the fluid properties in practical heat-transfer computations are determined for its mean (arithmetic mean) temperature. At the same time, the wall temperature is to be determined because of the approximate solution of the conjugate problem of convective heat transfer, hence, its selection as governing results in considerable difficulties.

To compare the result obtained with experiment, data obtained on pipes of sufficient length for known wall temperatures were selected. The open points in the Fig. 1 are values of $\langle Nu\rangle_f$ for aqueous solutions of glycerine of different concentrations [10] and of water [11], the blackened points are values of $\langle Nu(\infty)\rangle_f(\nu_f/\nu_w)^{-1/3}$ computed for the same tests. All the points lie in the Reynolds number range $Re_f < Re^* = 2460\ Pr_f^{-0.646}$ [4] in which the Nusselt heat transfer law is valid. Moreover, the condition $\xi > 0.05(\ln W)/3$ was confirmed. The wall temperature varied negligibly along the pipe length.

It is seen from the figure that the influence of the variable viscosity on the heat transfer grows with the increase in the initial viscosity ν_0, which is apparently associated with the growth of the parameter b.

The values of $\langle Nu\rangle_f(\nu_f/\nu_w)^{-1/3}$ lie considerably nearer to the quantity 1.88 than the corresponding values of $\langle Nu\rangle_f$. The rms deviation is 8% in the first case and 32.4% in the second.

Therefore, the influence of variable viscosity on the intensity of heat transfer in a laminar film is noticeable and can be taken into account by (21).

NOTATION

$Re = 4\Gamma/\nu = 4R$, Reynolds criterion; $Pe = \Gamma/a$, Peclet criterion; $Pr = \nu/a$, Prandtl criterion; $Nu = \alpha\delta/\lambda$, $\langle Nu\rangle = \langle\alpha\rangle\delta/\lambda$, local and mean Nusselt criterion; Γ, volume irrigation density, m^2/sec; b, $B = b\Delta t$, dimensional (deg^{-1}) and dimensionless coefficients taking account of the temperature dependence of the viscosity; c, specific heat of the fluid, J/kg·deg; g, free-fall acceleration, m/sec^2; $h = \delta/\delta_0$, dimensionless flux thickness; $\Delta t = \max|t - t_0|$, maximum temperature drop, deg; $T = (t - t_0)/\Delta t$, dimensionless temperature; T_m, mean fluid temperature over the layer cross section; u and v, velocity components along the x and y axes, m/sec; u_0, mean isothermal-flux velocity (for $x < 0$), m/sec; α, $\langle\alpha\rangle$, local and mean heat-transfer coefficients, $W/m^2\cdot deg$; β_j, eigenvalues of the linear problem; δ, layer thickness, m; λ, fluid coefficient of heat conductivity, W/m·deg; μ, dynamic fluid viscosity, kg/m·sec; ν, kinematic viscosity, m^2/sec; $\theta = (t - t_0)/\Delta t$, dimensionless wall temperature; $\xi = x/3Pe\delta_0$, $\eta = y/\delta_0 h$, dimensionless coordinates. Subscripts: f, fluid parameters at the mean-mass temperature; w, fluid parameters at the wall temperature; 0, fluid parameters for an isothermal flux ($x < 0$).

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NATURAL CONVECTIVE HEAT EXCHANGE BETWEEN ISOTHERMAL CONCENTRIC SPHERES

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The problem of natural convection in spherically concentric layers is considered. The heat-exchange similarity equation obtained agrees satisfactorily with the experimental data of [5].

At the present time there is great interest in analytical and numerical methods of solving natural convective problems in finite volumes. The majority of studies consider planar problems, with a minority devoted to cylindrical layers, while [1-3] consider spherical layers. A bibliography of the first two types of problem is presented in [4]. In [1] the authors consider natural convection of a viscous compressible gas (air, P = 0.714) for outer/inner diameter ratios in the range $1.1 \leqslant d_2/d_1 \leqslant 6$ and Grashof numbers from 10^3 to 10^6. Lawrence et al. [2] considered natural convection of incompressible air at low Rayleigh numbers. The authors attempted to fill a gap in theory for this region, but comparison of their results with the experiments of Bishop et al. [5] indicates a lack of success. In [3] (where in contrast to [1, 2] the exterior sphere was the hotter), natural convection of a compressible gas (air, P = 0.71) was considered. The heat-exchange similarity equation obtained in [3] was compared with the results of [5] and good agreement was found.

In the experimental study [5] a generalized heat-exchange equation was obtained for calculation of average heat liberation in spherical isothermal concentric layers for a wide range of Rayleigh numbers (determined by width of the layer) and Prandtl numbers (P = 4.7-4148; $Ra = 1.3 \cdot 10^3 - 5.8 \cdot 10^8$, D/d = 1.09-2.81).

But, it is often of importance to know such local characteristics as the velocity field, the temperature in the layer, the character of liquid motion, and local thermal fluxes (which are often quite complex), which at present cannot be experimentally determined. These difficulties may be avoided by numerical solution of the problem. Moreover, [1-3] considered only air, reducing the range of application of the analytical and numerical results obtained for calculation of natural convective heat exchange in spherical concentric liquid (gas) layers, the thermophysical characteristics of which differ from air. Therefore, in order to obtain a solution over a broadened range of Prandtl and Rayleigh numbers, an attempt was made to numerically solve the problem of natural convection in spherical concentric layers of both gases and liquids. Prandtl and Rayleigh numbers were varied over the range P = 0.2-5, Ra_d =

TABLE 1. Coefficients of Eq. (1)

φ	a_φ	b_φ	c_φ	d_φ
$\frac{\omega}{R\sin\beta}$	$\frac{R^2\sin^2\beta}{GP^2}$	$\frac{R^2\sin^2\beta}{GP}$	1	$R\sin\beta\left(\sin\beta\frac{\partial T}{\partial R}+\frac{\cos\beta}{R}\frac{\partial T}{\partial\beta}\right)$
ψ	0	$\frac{1}{R^2\sin^2\beta}$	1	$-\frac{\omega}{R^2\sin^2\beta}$
T	1	1	1	0

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